COMMON PRE-BOARD EXAMINATION 2022-23

Subject: MATHEMATICS (041)

Class: XII　　　　　　　　　　　　　　　　　　　　Time: 3 Hours
Date:　　　　　　　　　　　　　　　　　　　　　　Max. Marks: 80

General Instructions:

1. *This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.*
2. *Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.*
3. *Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.*
4. *Section C has 6 Short Answer (SA)-type questions of 3 marks each.*
5. *Section D has 4 Long Answer (LA)-type questions of 5 marks each.*
6. *Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.*

Q.No.		Marks
	SECTION – A **(Section A consists of 20 questions of 1 mark each)**	
1.	A be a non singular square matrix of order 3×3.Then$\|adjA\|$ is equal to (a)$\|A\|$　(b)3$\|A\|$　(c)$\|A\|^3$　(d))$\|A\|^2$	1
2.	If $A$is a 3× 3 matrix such that $\|A\|$=8, then $\|3A\|$ equals (a) 8　(b) 72　(c)216　(d) 24	1
3.	If $\vec{a}$ and $\vec{b}$ are unit vectors inclined at an angle θ, then the value of $\left\|\vec{a}-\vec{b}\right\|$ is (a) $2\, cos\frac{\theta}{2}$　(b)$2sin\frac{\theta}{2}$　(c)$2cos\theta$　(d) $2sin\theta$	1
4.	The value of $k(k<0)$ for which the function f defined as $f(x)=\begin{cases}\frac{1-coskx}{xsinx}, x\neq 0\\ \frac{1}{2}, x=0\end{cases}$ is continuous at $x=0$ is (a) -1　(b) ±1　(c) $\pm\frac{1}{2}$　(d $\frac{1}{2}$	1
5.	The least value of the function $f(x)=2cosx+x$ in the closed interval $\left[0,\frac{\pi}{2}\right]$ is : (a) 2　(b) $\frac{\pi}{6}+\sqrt{3}$　(c) $\frac{\pi}{2}$　(d) The least value does not exist	1
6.	Degree of the differential equation $[1+\left(\frac{dy}{dx}\right)^2]^{\frac{3}{2}}=\frac{d^2y}{dx^2}$ is (a) 2　(b)$\frac{3}{2}$　(c) not defined　(d) 4	1

7.	The direction cosines of a straight line, whose projection on the co-ordinate axes, OX, OY,OZ are 12,4,13 respectively,are (a) $\frac{12}{29},\frac{4}{29},\frac{13}{29}$ (b) $\frac{12}{\sqrt{329}},\frac{4}{\sqrt{329}},\frac{13}{\sqrt{329}}$ (c) $\frac{1}{12},\frac{1}{4},\frac{1}{3}$ (d) $\frac{12}{329},\frac{4}{329},\frac{13}{329}$	1
8.	The vector equation of the straight line $\frac{1-x}{3}=\frac{y+1}{-2}=\frac{3-z}{-1}$ is (a) $\vec{r}=\left(\hat{\imath}-\hat{\jmath}+3\hat{k}\right)+\lambda\left(3\hat{\imath}+2\hat{\jmath}-\hat{k}\right)$ (b) $\vec{r}=\left(\hat{\imath}-\hat{\jmath}+3\hat{k}\right)+\lambda\left(3\hat{\imath}-2\hat{\jmath}-\hat{k}\right)$ (c) $\vec{r}=\left(\widehat{3\imath}-2\hat{\jmath}-\hat{k}\right)+\lambda\left(\hat{\imath}-\hat{\jmath}+3\hat{k}\right)$ (d) $\vec{r}=\left(3\hat{\imath}+2\hat{\jmath}-\hat{k}\right)+\lambda\left(\hat{\imath}-\hat{\jmath}+3\hat{k}\right)$	1
9.	The value of $\int\sqrt{4-x^2}dx$ is (a)None of these (b)$\frac{x}{2}\sqrt{4-x^2}+2sin^{-1}\frac{x}{2}+C$ (c)$x\sqrt{4-x^2}+sin^{-1}\frac{x}{2}+C$ (d) $\frac{1}{2}x\sqrt{4-x^2}-2sin^{-1}\frac{x}{2}+C$	1
10.	If $\begin{bmatrix} cos\propto & -sin\propto \\ sin\propto & cos\propto \end{bmatrix}$, and $A+A'=1$, if the value of $\propto$ is (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{3\pi}{2}$ (d) π	1
11.	For an objective function $Z=ax+by$, where $a,b>0$; the corner points of the feasible region determined by a set of constraints (linear inequalities) are (0,20), (10,10), (30,30) and (0,40). The condition on a and b such that the maximum Z occurs at both the points (30,30) and (0,40) is a) $b-3a=0$ b) $a=3b$ c) $a+2b=0$ (d) $2a-b=0$	1
12.	Domain of $cos^{-1}x$ is (a) $[-1,0]$ (b) $[0,1]$ (c) None of these (d) $[-1,1]$	1
13.	Let A be a square matrix of order 3. If $\|A\|=-2$,then the value of determinant of $\|A\|adjA$ is (a) 8 (b)-8 (c) -1 (d) 32	1
14.	Two numbers are selected at random from integers 1 to 9.If the sum is even, what is the probability that both numbers are odd? (a) $\frac{5}{80}$ (b) $\frac{1}{6}$ (c) $\frac{4}{9}$ (d) $\frac{2}{3}$	1
15.	What is the equation of a curve passing through$(0,1)$ and whose differential equation is given by $dy=y\tan x\,dx$? (a) $y=\sec x$ (c) $y=\sin x$ (c) $y=cosec\ x$ (d) $y=\cos x$	1
16.	Function $f(x)=2x^3-9x^2+12x+29$ is monotonically decreasing when (a) $x>2$ (b) $1<x<2$ (c) $x<2$ (d) $x>3$	1
17.	$\int sin^3(2x+1)dx=?$ (a) $\frac{1}{2}\cos(2x+1)+\frac{1}{3}\ cos^3(2x+1)+C$ (b) $-\frac{1}{2}\cos(2x+1)+\frac{1}{6}\ cos^3(2x+1)+C$ (c) $\frac{1}{8}sin^4(2x+1)+C$ (d) None of these	1

18.	The area enclosed by the circle $x^2 + y^2 = 2$ is equal to (a) $4\pi^2\ sq\ units$ (b) $4\pi\ sq\ units$ (c) $2\pi\ sq\ units$ (d) $2\sqrt{2}\,\pi\ sq\ units$	1
	ASSERTION –REASON BASED QUESTIONS In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices. (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is not the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true.	
19.	**Assertion** (A) : If manufacturer can sell x items at a price of ₹$\left(5 - \frac{x}{100}\right)$ each. Then cost price of x items is ₹$\left(\frac{x}{5} + 500\right)$. Then, the number of items he should sell to earn maximum profit is 240 items. **Reason** (R) : The profit for selling x items is given by $\frac{24}{5}x - \frac{x^2}{100} - 300$	1
20.	**Assertion** (A) : The matrix $A = \begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}$ is singular. **Reason** (R) : A square matrix A is said to be singular, if $\|A\| = 0$	1
	<u>SECTION-B</u> **(Section B consists of 5 questions of 2 marks each)**	
21.	Write the cofactor of $\begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix}$ **OR** Solve the system of equations by matrix method $8x + 4y + 3z = 18$ $2x + y + z = 5$ $x + 2y + z = 5$	2
22.	Find the value of the $tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + tan^{-1}[\sin(\frac{-\pi}{2})]$	2
23.	Find $\frac{dy}{dx}$ if $y = x^{\sin x} + (\sin x)^x$ **OR** Find $\frac{dy}{dx}$ if $y^x = x^y$	2
24.	Given the probability that A can solve a problem is $\frac{2}{3}$, and the probability B can solve the same problem is $\frac{3}{5}$, find the probability that at least one of A and B will solve the problem.	2
25.	For what value of λ are the vectors $\vec{a}$ and $\vec{b}$ perpendicular to each other where; $\vec{a} = \lambda\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $\vec{b} = 4\hat{\imath} - 9\hat{\jmath} + 2\hat{k}$	2

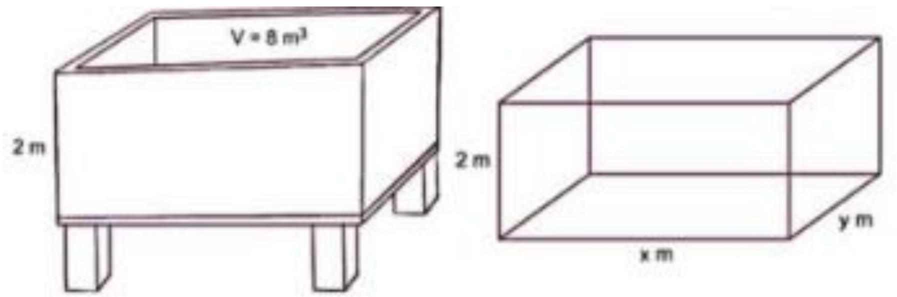
	SECTION-C **(Section C consists of 6 questions of 3 marks each)**	
26.	Find the particular solution of the differential equation $e^x\sqrt{1-y^2}dx + \frac{y}{x}dy = 0$ given that $y = 1$, when $x = 0$ **OR** Solve the differential equation $x\frac{dy}{dx} = x + y$	3
27.	Evaluate $\int_0^{\frac{\pi}{4}} \log(1 + \tan x)dx$	3
28.	Evaluate: $\int \frac{x^2+5x+3}{x^2+3x+2}dx$ **OR** Evaluate $\int \frac{x^2+x+1}{(x+2)(x^2+1)}dx$	3
29.	Find the particular solution: $x^2dy + (xy + y^2)\ dx = 0; y = 1$ when $x = 1$ **OR** Show that the differential equation $xcos\left(\frac{y}{x}\right)\frac{dy}{dx} = ycos\left(\frac{y}{x}\right) + x$ is homogeneous and solve it.	3
30.	Solve the following Linear Programming Problem graphically: Maximize Z=$600x + 400y$ subject to $x + 2y \leq 12, 2x + y \leq 12, x + \frac{5}{4}y \geq 5, x, y \geq 0$	3
31.	Show that the relation R in the set $A = \{x\epsilon Z : 0 \leq x \ \leq 12\}$, given by $R = \{(a, b): \|a - b\|$ is a multiple of 4$\}$ is an equivalence relation	3
	SETCION-D **(Section D consists of 4 questions of 5 marks each)**	
32.	Find the area of the region bounded by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$.	5
33.	Evaluate: $\int_1^4 \|x - 1\| + \|x - 2\| + \|x - 3\|dx$ **OR** Evaluate $\int_{-1}^2 \|x^3 - x\|\ dx$	5
34.	Find the shortest distance between the lines $\vec{r} = (1 - t)\hat{\imath} + (t - 2)\hat{\jmath}+(3-2t)\hat{k}$ $\vec{r} = (s + 1)\hat{\imath} + (2s - 1)\hat{\jmath} - (2s + 1)\hat{k}$ **OR** Show that the lines $\vec{r} = (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) + \lambda(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k})$ and $\vec{r} = (4\hat{\imath} + \hat{\jmath}) + \mu(5\hat{\imath} + 2\hat{\jmath} + \hat{k})$ intersect. Also, find their point intersection.	5
35.	$If\ x = sint$ and $y = \sin pt$, prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$	5

SECTION-E
(Case study based questions are compulsory)

36. CASE STUDY I — 4

Read the text carefully and answer the questions

On the request of villagers, a construction agency designs a tank with the help of an architect. Tank consists of a rectangular base with rectangular sides, open at the top so that its depth is 2m and volume 8m^3 as shown below. The construction of the tank costs ₹ 70 per sq.metre for the base and ₹ 45 per sq. metre for sides.



(i) Express making cost C in terms of length of rectangle

(ii) If x and y represent the length and breadth of its rectangular base, then find the relation between the variables

(iii) Find the value of x so that the cost of construction is minimum

OR

Verify by second derivative test that cost is minimum at a critical point.

37. CASE STUDY II — 4

Read the text carefully and answer the questions:

Three car dealers, say A,B and C, deals in three types of cars,namely Hatchback cars,Sedan cars, SUV cars. The sales figure of 2019 and 2020 showed that dealer A sold 120 Htachback,50 Sedan,10 SUV cars in 2019 and 300 Hatchback,150 Sedan,20 SUV cars in 2020; dealer B sold 100 Hatchback,30 Sedan,5 SUV cars in 2019 and 200 Hatchback,50 Sedan,6 SUV cars in 2020; dealer C sold 90 Hatchback,40 Sedan,2 SUV cars in 2019 and 100 Hatchback,60 Sedan,5 SUV cars in 2020.

(i) Write the matrix summarizing sales data of 2019 and 2020

(ii) Find the matrix summarizing sales data of 2020.

(iii) Find the total number of cars sold in two given years, by each dealer?

OR

If each dealer receives a profit of ₹50000 on sale of a Hatchback ₹100000 on sale of a Sedan and ₹200000 on sale of an SUV, then find the amount of profit received in the year 2020 by each dealer.

38.	CASE STUDY III Read the text carefully and answer the questions: To teach the application of probability a maths teacher arranged a surprise game for 5 of his students namely Govind, Girish, Vinod, Abhishek and Ankit. He took a bowl containing tickets numbered 1 to 50 and told the students go one by one and draw two tickets simultaneously from the bowl and replace it after noting the numbers.  (i) Teacher ask Govind, what is the probability that tickets are by Abhishek, shows a prime number on one ticket and a multiple of 4 on other ticket? (ii) Teacher ask Girish, what is the probability that tickets drawn by Ankit, shows an even number on first ticket and an odd number on second ticket?	4

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